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Matching the observational value of the cosmological constant[1]

E. Elizalde[2]

Instituto de Ciencias del Espacio (CSIC)

& Institut d'Estudis Espacials de Catalunya (IEEC/CSIC)

Edifici Nexus, Gran Capità 2-4, 08034 Barcelona, Spain

and

Departament ECM i IFAE, Facultat de Física,

Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain

Abstract

A simple model is introduced in which the cosmological constant is interpreted as a true Casimir effect on a scalar field filling the universe (e.g. $\mathbf{R} \times \mathbf{T^p} \times \mathbf{T^q}$, $\mathbf{R} \times \mathbf{T^p} \times \mathbf{S^q}, \ldots$). The effect is driven by compactifying boundary conditions imposed on some of the coordinates, associated with large and with small scales (the total number of large spatial coordinates being always three). The very small —but non zero— value of the cosmological constant obtained from recent astrophysical observations can be perfectly matched with the results coming from the model, by just fixing the numbers of —actually compactified— ordinary and tiny dimensions to be very common ones, and being the compactification radius (for the last) in the range $(1 - 10^3)\ l_{Pl}$, where l_{Pl} is the Planck length. This corresponds to solving, in a way, what has been termed by Weinberg the *new* cosmological constant problem. Moreover, a marginally closed universe is favored by the model, again in coincidence with independent analysis of the observational results.

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[2]Presently on leave at Department of Mathematics, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139. E-mail: elizalde@math.mit.edu elizalde@ieec.fcr.es http://www.ieec.fcr.es/recerca/cme/eli.html

1 Introduction

The issue of the cosmological constant has got renewed thrust from the observational evidence of an acceleration in the expansion of our Universe, recently reported by two different groups [1, 2]. There has been some controversy on the reliability of the results obtained from those observations and on its precise interpretation, by a number of different reasons [3, 4]. Anyway, there is presently reasonable consensus among the community of cosmologists that it certainly could happen that there is, in fact, an acceleration, and that it has the order of magnitude obtained in the above mentioned observations. In support of this consensus, the recently issued analysis of the data taken by the BOOMERANG [5] and MAXIMA-1 [6] balloons have been correspondigly crossed with those from the just mentioned observations, to conclude that the results of BOOMERANG and MAXIMA-1 can perfectly account for an accelerating universe and that, taking together both kinds of observations, one inferes that we most probably live in a flat universe. As a consequence, many theoretists have urged to try to explain this fact, and also to try to reproduce the precise value of the cosmological constant coming from these observations, in the available models [7, 8, 9].

Now, as crudely stated by Weinberg in a recent review [10], it is even more difficult to explain why the cosmological constant is so small but non-zero, than to build theoretical models where it exactly vanishes [11]. Rigorous calculations performed in quantum field theory on the vacuum energy density, ρ_V, corresponding to quantum fluctuations of the fields we observe in nature, lead to values that are over 120 orders of magnitude in excess of the values allowed by observations of the space-time around us.

Rather than trying to understand the fine-tuned cancellation of such enormous values at this *local* level (a very difficult question that we are going to leave unanswered, and even unattended, here), in this paper we will elaborate on a quite simple and primitive idea (but, for the same reason, of far reaching, inescapable consequences), related with the *global* topology of the universe [12] and in connection with the possibility that a very faint, massless scalar field pervading the universe could exist. Fields of this kind are ubiquitous in inflationary models, quintessence theories, and the like. In other words, we do not pretend here to solve the old problem of the cosmological constant, not even to contribute significantly to its understanding, but just to present an extraordinarily simple model which shows that the right order of magnitude of (some contributions to) ρ_V, in the precise range deduced from the astrophysical observations [1, 2], e.g. $\rho_V \sim 10^{-10}$ erg/cm^3, are not difficult to obtain. To say it in different words, we only address here what has been termed by Weinberg [10] the *new* cosmological constant problem.

In short, we shall assume the existence of a scalar field background extending throught the universe and shall calculate the contribution to the cosmological constant coming from the Casimir energy density [13] corresponding to this field for some typical boundary conditions. The ultraviolet contributions will be safely set to zero by some mechanism of a fundamental theory. Another hypothesis will be the existence of both large and small dimensions (the total number of large spatial coordinates will be always three), some of which (from each class) may be compactified, so that the global topology of the universe will play an important role, too. There is by now a quite extense literature both in the subject of what is the global topology of spatial sections of the universe [12] and also on the issue of the possible contribution of the Casimir effect as a source of some sort of cosmic energy, as in the case of the creation of a neutron star [14]. There are arguments that favor different topologies, as a compact hiperbolic manifold for the spatial section, what would have clear observational consequences [15]. Other interesting work along these lines was reported in [16] and related ideas have been discussed very recently in [17]. However, our paper differs from all those in several respects. To begin, the emphasis is put now in obtaining the right order of magnitude for the effect, e.g., one that matches the recent observational results. At the present stage, in view of the observational precission, it has no sense to consider the whole amount of possibilities concerning the nature of the field, the different models for the topology of the universe, and the different boundary conditions possible.

At this level, from our previous experience in these calculations and from the many tables (see, e.g., [18, 19, 20] where precise values of the Casimir effect corresponding to a number of different configurations have been reported), we realize that the range of *orders of magnitude* of the vacuum energy density for the most common possibilities is not so widespread, and may only differ by at most a couple of digits. This will allow us, both for the sake of simplicity *and* universality, to deal with a most simple situation, which is the one corresponding to a scalar field with periodic boundary conditions. Actually, as explained in [21] in detail, all other cases for parallel plates, with any of the usual boundary conditions, can be reduced to this one, from a mathematical viewpoint.

2 Two basic space-time models

Let us thus consider a universe with a space-time of one of the following types: $\mathbf{R^{d+1}} \times \mathbf{T^p} \times \mathbf{T^q}$, $\mathbf{R^{d+1}} \times \mathbf{T^p} \times \mathbf{S^q}, \ldots$, which are actually plausible models for the space-time topology.

A (nowadays) free scalar field pervading the universe will satisfy

$$(-\Box + M^2)\phi = 0, \tag{1}$$

restricted by the appropriate boundary conditions (e.g., periodic, in the first case considered). Here, $d \geq 0$ stands for a possible number of non-compactified dimensions.

Recall now that the physical contribution to the vacuum or zero-point energy $< 0|H|0 >$ (where H is the Hamiltonian corresponding to our massive scalar field and $|0>$ the vacuum state) is obtained on subtracting to these expression —with the vacuum corresponding to our compactified spatial section with the assumed boundary conditions— the vacuum energy corresponding to the same situation with the only change that the compactification is absent (in practice this is done by conveniently sending the compactification radii to infinity). As well known, both of these vacuum energies are in fact infinite, but it is its *difference*

$$E_C = < 0|H|0 >|_R - < 0|H|0 >|_{R\to\infty} \tag{2}$$

(where R stands here for a typical compactification length) that makes physical sense, giving rise to the finite value of the Casimir energy E_C, which will depend on R (after a well defined regularization/renormalization procedure is carried out). In fact we will discuss the Casimir (or vacuum) energy *density*, $\rho_C = E_C/V$, which can account for either a finite or an infinite volume of the spatial section of the universe (from now on we shall assume that all diagonalizations already correspond to energy densities, and the volume factors will be replaced at the end). In terms of the spectrum $\{\lambda_n\}$ of H:

$$< 0|H|0 >= \frac{1}{2}\sum_n \lambda_n, \tag{3}$$

where the sum over n is a sum over the whole spectrum, which involves, in general, several continuum and several discrete indices. The last appear tipically when compactifying the space coordinates (much in the same way as time compactification gives rise to finite-temperature field theory), as in the cases we are going to consider. Thus, the cases treated will involve integration over d continuous dimensions and multiple summations over $p + q$ indices (for a pedagogical description of this procedure, see [21]).

To be precise, the physical vacuum energy density corresponding to our case, where the contribution of a scalar field, ϕ in a (partly) compactified spatial section of the universe is considered, will be denoted by ρ_ϕ (note that this is just the contribution to ρ_V coming from this field, there might be other, in general). It is given by

$$\rho_\phi = \frac{1}{2}\sum_{\mathbf{k}} \frac{1}{\mu}\left(k^2 + M^2\right)^{1/2}, \tag{4}$$

where the sum $\sum_{\mathbf{k}}$ is a generalized one (as explained above) and μ is the usual mass-dimensional parameter to render the eigenvalues adimensional (we take $\hbar = c = 1$ and shall insert the dimensionfull units only at the end of the calculation). The mass M of the field will be here considered to be arbitrarily small and will be kept different from zero, for the moment, for computational reasons —as well as for physical ones, since a very tiny mass for the field can never be excluded. Some comments about the choice of our model are in order. The first seems obvious: the coupling of the scalar field to gravity should be considered. This has been done in all detail in, e.g., [22] (see also the references therein). The conclusion is that taking it into account does not change the results to be obtained here. Of course, the renormalization of the model is rendered much more involved, and one must enter a discussion on the orders of magnitude of the different contributions, which yields, in the end, an ordinary perturbative expansion, the coupling constant being finally re-absorbed into the mass of the scalar field. In conclusion, we would not gain anything new by taking into account the coupling of the scalar field to gravity. Owing, essentially, to the smallness of the resulting mass for the scalar field, one can prove that, quantitatively, the difference in the final result is at most of a few percent.

Another important consideration is the fact that our model is stationary, while the universe is expanding. Again, careful calculations show that this effect can actually be dismissed at the level of our order of magnitude calculation, since its value cannot surpass the one that we will get (as is seen from the present value of the expansion rate $\Delta R/R \sim 10^{-10}$ per year or from direct consideration of the Hubble coefficient). As before, for the sake of simplicity, and in order to focus just on the essential issues of our argument, we will perform a (momentaneously) static calculation. As a consequence, the value of the Casimir energy density, and of the cosmological constant, to be obtained will correspond to the present epoch, and are bound to change with time.

The last comment at this point would be that (as shown by the many references mentioned above), the idea presented here is not entirely new. However, the simplicity and the generality of its implementation below are indeed brand new. The issue at work here is absolutely independent of *any* specific model, the only assumptions having been clearly specified before (e.g., existence of a very light scalar field and of some reasonably compactified scales, see later). Secondly, it will turn out, in the end, that the only 'free parameter' to play with (the number of compactified dimensions) will actually not be that 'free' but, on the contray, very much constrained to have an admissible value. This will become clear after the calculations below. Thirdly, although the calculation may seem easy to do, in fact it is not so. Recently

derived reflection identities will allow us to to perform it *analitically*, for the first time.

3 The vacuum energy density and its regularization

To exhibit explicitly a couple of the wide family of cases considered, let us write down in detail the formulas corresponding to the two first topologies, as described above. For a (p,q)-toroidal universe, with p the number of 'large' and q of 'small' dimensions:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j \prod_{h=1}^q b_h} \int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^\infty \sum_{\mathbf{m}_q=-\infty}^\infty \left[\sum_{j=1}^p \left(\frac{2\pi n_j}{a_j}\right)^2 + \sum_{h=1}^q \left(\frac{2\pi m_h}{b_h}\right)^2 + M^2 \right]^{1/2} \quad (5)$$

$$\sim \frac{1}{a^p b^q} \sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^\infty \left(\frac{1}{a^2}\sum_{j=1}^p n_j^2 + \frac{1}{b^2}\sum_{h=1}^q m_h^2 + M^2 \right)^{(d+1)/2+1}, \quad (6)$$

where the last formula corresponds to the case when all large (resp. all small) compactification scales are the same. In this last expression the squared mass of the field should be divided by $4\pi^2\mu^2$, but we have renamed it again M^2 to simplify the ensuing formulas (as M is going to be very small, we need not keep track of this change). We also will not take care for the moment of the mass-dim factor μ in other places —as is usually done— since formulas would get unnecesarily complicated and there is no problem in recovering it at the end of the calculation. For a (p-toroidal, q-spherical)-universe, the expression turns out to be

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j\ b^q} \int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^\infty \sum_{l=1}^\infty P_{q-1}(l) \left[\sum_{j=1}^p \left(\frac{2\pi n_j}{a_j}\right)^2 + \frac{Q_2(l)}{b^2} + M^2 \right]^{1/2} \quad (7)$$

$$\sim \frac{1}{a^p b^q} \sum_{\mathbf{n}_p=-\infty}^\infty \sum_{l=1}^\infty P_{q-1}(l) \left(\frac{4\pi^2}{a^2}\sum_{j=1}^p n_j^2 + \frac{l(l+q)}{b^2} + M^2 \right)^{(d+1)/2+1}, \quad (8)$$

where $P_{q-1}(l)$ is a polynomial in l of degree $q-1$, and where the second formula corresponds to the similar situation as the second one before. On dealing with our observable universe, in all these expression we assume that $d = 3-p$, the number of non-compactified, 'large' spatial dimensions (thus, no d dependence will remain).

As is clear, all these expressions for ρ_ϕ need to be regularized. We will use zeta function regularization, taking advantage of the very powerful equalities that have been derived recently [23, 24], which reduce the enormous burden of such computations to the easy application of some formulas. For the sake of completeness, let us very briefly summarize how this works [25, 21]. We deal here only with the case when the spectrum of the Hamiltonian operator is known explicitly. Going back to the most general expressions of the Casimir

energy corresponding to this case, namely Eqs. (??)-(4), we replace the exponents in them with a complex variable, s, thus obtaining the zeta function associated with the operator as:

$$\zeta(s) = \frac{1}{2} \sum_{\mathbf{k}} \left(\frac{k^2 + M^2}{\mu^2} \right)^{-s/2} . \tag{9}$$

The next step is to perform the analytic continuation of the zeta function from a domain of the complex s-plane with Re s big enough (where it is perfectly defined by this sum) to the point $s = -1$, to obtain:

$$\rho_\phi = \zeta(-1). \tag{10}$$

The effectiveness of this method has been sufficiently described before (see, e.g., [18, 19]). As we know from precise Casimir calculations in those references, no further subtraction or renormalization is needed in the cases here considered, in order to obtain the physical value for the vacuum energy density (there is actually a subtraction at infinity taken into account, as carefully described above, but it is of null value, and no renormalization, not even a finite one, very common to other frameworks, applies here).

Using the recent formulas [23] that generalize the well-known Chowla-Selberg expression to the situations considered above, Eqs. (5) and (7) —namely, multidimensional, massive cases— we can provide arbitrarily accurate results for different values of the compactification radii. However, as argued above we can only aim here at matching the *order of magnitude* of the Casimir value and, thus, we shall just deal with the most simple case of Eq. (6) (or (5), which yield the same orders of magnitude as the rest). Also in accordance with this observation, we notice that among the models here considered and which lead to the values that will be obtained below, there are in particular the very important typical cases of isotropic universes with the spherical topology. As all our discussion here is in terms of orders of magnitude and not of precise values with small errors, all these cases are included on *equal footing*. But, on the other hand, it has no sense to present a lengthy calculation dealing in detail with all the possible spatial geometries. Anyhow, all these calculations are very similar to the one to be carried out here, as has been described in detail elsewhere [16, 18, 19].

For the analytic continuation of the zeta function corresponding to (5), we obtain [23]:

$$\begin{aligned} \zeta(s) &= \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2} b^{q-(s-1)/2} \Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^h n_j^2}{\sum_{k=1}^q m_k^2 + M^2} \right)^{(s-1)/4} \\ & \quad \times K_{(s-1)/2} \left[\frac{2\pi a}{b} \sqrt{\sum_{j=1}^h n_j^2 \left(\sum_{k=1}^q m_k^2 + M^2 \right)} \right], \end{aligned} \tag{11}$$

where $K_\nu(z)$ is the modified Bessel function of the second kind. Having performed already the analytic continuation, this expression is ready for the substitution $s = -1$, and yields

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sqrt{\frac{\sum_{k=1}^q m_k^2 + M^2}{\sum_{j=1}^h n_j^2}}\; K_1 \left[\frac{2\pi a}{b} \sqrt{\sum_{j=1}^h n_j^2 \left(\sum_{k=1}^q m_k^2 + M^2 \right)} \right] . \quad (12)$$

Now, from the behaviour of the function $K_\nu(z)$ for small values of its argument,

$$K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0, \tag{13}$$

we obtain, in the case when M is very small,

$$\begin{aligned} \rho_\phi \;\; &= \;\; -\frac{1}{a^p b^{q+1}} \left\{ M\, K_1 \left(\frac{2\pi a}{b} M \right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^h n_j^2}}\; K_1 \left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^h n_j^2} \right) \right. \\ &\quad \left. + \,\mathcal{O} \left[q\sqrt{1+M^2} K_1 \left(\frac{2\pi a}{b} \sqrt{1+M^2} \right) \right] \right\} . \end{aligned} \tag{14}$$

At this stage, the only presence of the mass-dim parameter μ is as M/μ everywhere. This does not conceptually affect the small-M limit, $M/\mu << b/a$. Using (13) and inserting now in the expression the $\hbar$ and c factors, we finally get

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q} \left[1 + \sum_{h=0}^{p} \binom{p}{h} 2^h \alpha \right] + \mathcal{O} \left[q K_1 \left(\frac{2\pi a}{b} \right) \right], \tag{15}$$

where α is some finite constant, computable and under control, which is obtained as an explicit geometrical sum in the limit $M \to 0$. It is remarkable that we here obtain such a well defined limit, independent of M^2, provided that M^2 is small enough. In other words, a physically very nice situation turns out to correspond, precisely, to the mathematically rigorous case. This is moreover, let me repeat, the kind of expression that one gets not just for the model considered, but for *many* general cases, corresponding to different fields, topologies, and boundary conditions —aside from the sign in front of the formula, that may change with the number of compactified dimensions and the nature of the boundary conditions (in particular, for Dirichlet boundary conditions one obtains a value in the same order of magnitude but of opposite sign).

4 Numerical results

For the most common variants, the constant α in (15) has been calculated to be of order 10^2, and the whole factor, in brackets, of the first term in (15) has a value of order 10^7. This shows the value of a precise calculation, as the one undertaken here, together with the fact that

just a naive consideration of the dependences of ρ_ϕ on the powers of the compactification radii, a and b, is *not enough* in order to obtain the correct result. Notice, moreover, the non-trivial change in the power dependencies from going from Eq. (14) to Eq. (15).

For the compactification radii at small scales, b, we shall simply take the magnitude of the Planck length, $b \sim l_{P(lanck)}$, while the typical value for the large scales, a, will be taken to be the present size of the observable universe, $a \sim R_U$. With this choice, the order of the quocient a/b in the argument of K_1 is as big as: $a/b \sim 10^{60}$. Thus, we see immediately that, in fact, the final expression for the vacuum energy density is completely independent of the mass M of the field, provided this is very small (eventually zero). In fact, since the last term in Eq. (15) is exponentially vanishing, for large arguments of the Bessel function K_1, this contribution is zero, for all practical purposes, what is already a very nice result. Taken in ordinary units (and after tracing back all the transformations suffered by the mass term M) the actual bound on the mass of the scalar field is $M \leq 1.2 \times 10^{-32}$ eV, that is, physically zero, since it is lower by several orders of magnitude than any bound comming from the more usual SUSY theories −where in fact scalar fields with low masses of the order of that of the lightest neutrino do show up [8], which may have observable implications.

ρ_ϕ	$p=0$	$p=1$	$p=2$	$p=3$
$b = l_P$	10^{-13}	10^{-6}	1	10^5
$b = 10\, l_P$	10^{-14}	(10^{-8})	10^{-3}	10
$b = 10^2 l_P$	10^{-15}	$[10^{-10}]$	10^{-6}	10^{-3}
$b = 10^3 l_P$	10^{-16}	(10^{-12})	$[10^{-9}]$	10^{-7}
$b = 10^4 l_P$	10^{-17}	10^{-14}	(10^{-12})	$[10^{-11}]$
$b = 10^5 l_P$	10^{-18}	10^{-16}	10^{-15}	10^{-15}

Table 1: Orders of magnitude of the vacuum energy density contribution, ρ_ϕ, of a massless scalar field to the cosmological constant, ρ_V, for p large compactified dimensions and $q = p+1$ small compactified dimensions, $p = 0, \ldots, 3$, for different values of the small compactification length, b, proportional to the Planck length l_P. In brackets are the values that exactly match the observational value of the cosmological constant, and in parenthesis the otherwise closest approximations to that value.

By replacing all these values in Eq. (15), we obtain the results listed in Table 1, for the orders of magnitude of the vacuum energy density corresponding to a sample of different numbers of compactified (large and small) dimensions and for different values of the small

compactification length in terms of the Planck length. Notice again that the total number of large space dimensions is three, as corresponds to our observable universe. As we see from the table, good coincidence with the observational value for the cosmological constant is obtained for the contribution of a massless scalar field, ρ_ϕ, for p large compactified dimensions and $q = p + 1$ small compactified dimensions, $p = 0, \dots, 3$, and this for values of the small compactification length, b, of the order of 100 to 1000 times the Planck length l_P (what is actually a very reasonable conclusion, according also to other approaches). To be noticed is the fact that full agreement is obtained only for cases where there is exactly one small compactified dimension in excess of the number of large compactified dimensions. We must point out that the p large and q small dimensions are not all that are supposed to exist (in that case p should be at least, and at most, 3 and the other cases would lack any physical meaning). In fact, as we have pointed out before, p and q refer to the compactified dimensions only, but there may be other, non-compactifed dimensions (exactly $3 - p$ in the case of the 'large' ones), what translates into a slight modification of the formulas above, but does not change the order of magnitude of the final numbers obtained, assuming the most common boundary conditions for the non-compactified dimensions (see e.g. [19] for an explanation of this technical point). In particular, the cases of pure spherical compactification and of mixed toroidal (for small magnitudes) and spherical (for big ones) compactification can be treated in this way and yield results in the same order of magnitude range. Both these cases correspond to (observational) isotropic spatial geometries. Also to be remarked again is the non-triviality of these calculations, when carried out exactly, as done here, to the last expression, what is apparent from the use of the generalized Chowla-Selberg formula. Simple power counting is absolutely unable to provide the correct order of magnitude of the results.

5 Conclusions

Dimensionally speaking, within the global approach adopted in the present paper everything is dictated, in the end, by the two basic lengths in the problem, which are its Planck value and the radius of the observable Universe. Just by playing with these numbers in the context of our (very precise) calculation of the Casimir effect, we have shown that the observed value of ρ_V may be remarkably well fitted, under general hypothesis, for the most common models of the space-time topology. Notice also that the most precise fits with the observational value of the cosmological constant are obtained for b between $b = 100\, l_P$ and $b = 1000\, l_P$, with (1,2) and (2,3) compactified dimensions, respectively. The fact that the value obtained

for the cosmological constant is so sensitive to the input may be viewed as a drawback but also, on the contrary, as a very *positive* feature of our model. For one, the table has a sharp discriminating power. In other words, there is in fact no tuning of a 'free parameter' in our model and the number of large compactified dimensions could have been fixed beforehand, to respect what we know already of our observable universe.

Also, it proves that the observational value is not easy at all to obtain. The table itself proves that there is only very little chance of getting the right figure (a truly narrow window, since very easily we are off by several orders of magnitude). In fact, if we trust this value with the statistics at hand, we can undoubtedly claim −through use of our model− that the ones so clearly picked up by Table 1 are *the* only two possible configurations of our observable universe (together with a couple more coming from corresponding spherical compactifications). And all them correspond to a marginally closed universe, in full agreement too with other completely independent analysis of the observational data [4, 1, 2].

Many questions may be posed to the simple models presented here, as concerning the dynamics of the scalar field, its couplings with gravity and other fields, a possible non-symmetrical behaviour with respect to the large and small dimensions, or the relevance of vacuum polarization (see [26], concerning this last point). Above we have already argued that they can be proven to have little influence on the final numerical result (cf., in particular, the mass obtained for the scalar field in Ref. [22], extremely close to our own result, and the corresponding discussion there). From the very existence and specific properties of the cosmic microwave radiation (CMB) −which mimics somehow the situation described (the 'mass' corresponding to the CMB is also in the sub-lightest-neutrino range)− we are led to the conclusion that such a field could be actually present, unnoticed, in our observable universe. In fact, the existence of scalar fields of very low masses is also demanded by other frameworks, as SUSY models, where the scaling behaviour of the cosmological constant has been considered [8].

Let us finally recall that the Casimir effect is an ubiquitous phenomena. Its contribution may be small (as it seems to be the case, yet controverted, to sonoluminiscence), of some 10-30% (that is, of the right order of magnitude, as in wetting phenomena involving He in condensed matter physics), or even the whole thing (as in recent, dedicated experimental confirmations of the effect). Here we have seen that it provides a contribution of the right order of magnitude, corresponding to our present epoch in the evolution of the universe. The implication that this calculation bears for the early universe and inflation is not clear from the final result, since it should be adapted to the situation and boundary conditions

corresponding to those primeval epochs, what cannot be done straightforwardly. Work along this line is in progress.

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